# 様相論理による統計的因果の形式化

川本 裕輔　　佐藤 哲也　　末永 幸平

因果効果を表現し因果推論の要件を記述するための形式言語 StaCL (Statistical Causality Language) を提案する．具体的に，変数の間の因果的性質を論理式で表現するために，介入を表す様相演算子と因果述語の概念を導入する．また，介入や因果述語に関する公理を持つ StaCL の演繹体系を定義する．この演繹体系が統計的因果モデルに基づくクリプキモデルに対して健全であり，統計的因果推論における標準的な道具立てである Pearl の do 計算の規則を導出できる程度に表現力豊かであることを示す．最後に、因果推論の正しさを StaCL の論理式で定式化し説明できることを具体例で示す．

## 1 Introduction

*Statistical causality* has been gaining significant importance in a variety of research fields. In particular, in life sciences, more and more researchers have been using statistical techniques to discover *causal relationships* from experiments and observations. However, these statistical methods can easily be misused or misinterpreted. In fact, it is reported that many research articles have serious errors in the applications and interpretations of statistical methods [8] [27].

A common mistake is to misinterpret statistical *correlation* as statistical *causality*. Notably, when we analyze observational data without experimental interventions, we may overlook some requirements for causal inference and make wrong calculations, leading to incorrect conclusions about the causality.

For this reason, the scientific community has developed guidelines on many requirements for statistical analyses [37] [29]. However, since there is no formal language to describe the entire procedures and their requirements, we refer to guidelines manually and cannot formally guarantee the correctness of analyses.

To address these problems, we propose a logic-based approach to formalizing and explaining the correctness of statistical causal inference. Specifically, we introduce a formal language called *statistical causality language* (StaCL) to formally describe and check the requirements for statistical causal inference. We consider this work as the first step to building a framework for formally guaranteeing and explaining the reliability of scientific research.

**Contributions.** Our main contributions are as follows:

- We propose *statistical causality language* (StaCL) for formalizing and explaining statistical causality by using modal operators for in-

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Yusuke Kawamoto, 産業技術総合研究所, National Institute of Advanced Industrial Science and Technology.
Tetsuya Sato, 東京工業大学, Tokyo Institute of Technology.
Kohei Suenaga, 京都大学, Kyoto University.

terventions.

- We define a *Kripke model for statistical causality*. To formalize not only statistical correlation but also statistical causality, we introduce a *data generator* in a possible world to model a causal diagram in a Kripke model.
- We introduce the notion of *causal predicates* to express statistical causality and interpret them using a data generator instead of a valuation in a Kripke model. In contrast, *(classical) predicates* are interpreted using a valuation in a Kripke model to express only *statistical correlations.*
- We introduce a sound deductive system $\mathbf{AX}^{CP}$ for StaCL with axioms for probability distributions, interventions, and causal predicates. These axioms are expressive enough to reason about all causal effects identifiable by Pearl's *do-calculus* [30]. We show that $\mathbf{AX}^{CP}$ can reason about the correctness of causal inference methods (e.g., backdoor adjustment). Unlike prior work, $\mathbf{AX}^{CP}$ does not aim to conduct causal inference about a specific causal diagram; rather, it concerns the correctness of the inference methods for any diagram. To the best of our knowledge, ours appears to be the first modal logic that can specify and reason about the requirements for causal inference.

**Related Work.** Many studies on causal reasoning rely on causal diagrams [31]. Whereas they aim to reason about a specific diagram, our logic-based approach aims to specify and reason about the requirements for causal inference methods.

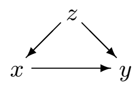
Logic-based approaches for formalizing causal reasoning have been proposed. To name a few, Halpern and Pearl provide logic-based definitions of actual causes where logical formulas with events formalize counterfactuals [12] [13] [11]. Probabilistic logical languages [19] are proposed to axiomatize causal reasoning with observation, intervention, and counterfactual inference. Unlike our logic, however, their framework does not aim to syntactically derive the correctness of statistical causal inference. The causal calculus [28] is used to provide a logical representation [4] [3] of Pearl [31]'s structural causal model. The counterfactual-observational language [1] can reason about interventionist counterfactuals and has an axiomatization that is complete w.r.t. a causal team semantics. A modal logic in [2] integrates causal and epistemic reasoning. While these works deal with deterministic cases only, our StaCL can reason about statistical causality in probabilistic settings.

There have been studies on incorporating probabilities into team semantics [15]. For example, team semantics is used to deal with the dependence and independence among random variables [6] [5]. A probabilistic team semantics is provided for a first-order logic that can deal with conditional independence [7]. A team semantics is also introduced for logic with exact/approximate dependence and independence atoms [14]. Unlike our StaCL, however, these works do not allow for deriving the do-calculus or the correctness of causal inference methods.

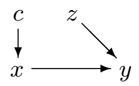
Concerning the axiomatic characterization of causality, Galles and Pearl [9] prove that the axioms of composition, effectiveness, and reversibility are sound and complete with respect to the structural causal models. They also show that the reversibility axiom can be derived from the composition axiom if the causal diagram is acyclic (i.e., has no feedback loop). Halpern [10] provides axiomatizations for more general classes of causal models with feedback and with equations that may have no solutions. In contrast, our deductive system $\mathbf{AX}^{CP}$ has axioms for causal predicates and two forms of interventions that can derive the rules of Pearl's do-calculus [30], while being equipped with axioms corresponding to the composition and effectiveness

Table1: Recovery rates of patients with/without taking a drug.

| | Drug $x=1$ | No-drug $x=0$ |
|---|---|---|
| Male | **0.90** (18/20) | 0.85 (68/80) |
| Female | **0.69** (55/80) | 0.60 (12/20) |
| Total | 0.73 (73/100) | **0.80** (80/100) |




(a) The actual diagram $G$ with a gender (confounder) $z$, a treatment $x$, and an outcome $y$.




(b) The diagram $G\lceil c/x \rceil$ with an intervention to $x$.

Fig.1: Causal diagrams in Example 1.

axioms mentioned above only for acyclic diagrams.

For the efficient computation of causal reasoning, constraint solving is applied [17] [18] [35]. Probabilistic logic programming is used to encode and reason about a specific causal diagram [32]. These are orthogonal to the goal of our work.

Finally, a few studies propose modal logic for statistical methods. Statistical epistemic logic [20] [21] [22] specifies various properties of machine learning. Belief Hoare logic [24] [26] can reason about statistical hypothesis testing programs. However, unlike our StaCL, these cannot reason about statistical causality.

## 2 Illustrating Example

We first present a simple example to explain our framework.

**Example 1 (Drug's efficacy)** We attempt to check a drug's efficacy for a disease by observing a situation where some patients take a drug and the others do not.

Table 1 shows the recovery rates and the numbers of patients treated with/without the drug. For both males and females, *more* patients recover by taking the drug. However, for the combined population, the recovery rate with the drug (0.73) is *less* than that without it (0.80). This inconsistency is called *Simpson's paradox* [34], showing the difficulty of identifying causality from observed data.

To model this, we define three variables: a *treatment* $x$ (1 for drug, 0 for no-drug), an *outcome* $y$ (1 for recovery, 0 for non-recovery), and a gender $z$. Fig. 1a depicts their causal dependency; the arrow $x \rightarrow y$ denotes that $y$ depends on $x$. The *causal effect* $p(y|do(x=c))$ of a treatment $x=c$ on an outcome $y$ [31] is defined as the distribution of $y$ in case $y$ were generated from $x=c$ (Fig. 1b).

However, since the gender $z$ influences the choice of the treatment $x$ in reality (Fig. 1a), the causal effect $p(y|do(x=c))$ depends on the common cause $z$ of $x$ and $y$ and differs from the correlation $p(y|x=c)$. Indeed, in Table 1, 80 % of females chose to take the drug ($x = 1$) while only 20 % of males did so; this dependency of $x$ on the gender $z$ leads to Simpson's paradox in Table 1. Thus, calculating the causal effect requires an "adjustment" for $z$, as explained below.

**Overview of the Framework.** We describe reasoning about the causal effect in Example 1 using logical formulas in our formal language StaCL (Section 5).

We define $\varphi_{\mathrm{RCT}} \stackrel{\mathrm{def}}{=} \lceil c/x \rceil (c_0 = y)$ to express a *randomized controlled trial (RCT)*, where we randomly divide the patients into two groups: one taking the drug ($x = 1$) and the other not ($x = 0$). This random choice of the treatment $x$ is expressed by the intervention $\lceil c/x \rceil$ for $c = 0, 1$ in the diagram $G\lceil c/x \rceil$ (Fig. 1b). Since $x$ is independent of $z$ in $G\lceil c/x \rceil$, the causal effect $p(y|do(x=c))$ of $x$ on the outcome $y$ is given as $y$'s distribution $c_0$ observed in the experiment in $G\lceil c/x \rceil$.

In contrast, $\varphi_{\mathrm{BDA}} \stackrel{\mathrm{def}}{=} (f = y|_{z,x=c} \wedge c_1 = z \wedge c_0 = f(c_1) \downarrow_y)$ describes the inference about the causal effect from observation *without* intervention to $x$ (Fig. 1a). This saves the cost of the experiment and avoids ethical issues in random treatments. Instead, to avoid Simpson's paradox, the inference $\varphi_{\mathrm{BDA}}$ conducts a *backdoor adjustment* (Section 7) to cope with the confounder $z$.

Concretely, the backdoor adjustment $\varphi_{\mathrm{BDA}}$ computes $x$'s causal effect on $y$ as follows. We first obtain the conditional distribution $f \stackrel{\mathrm{def}}{=} y|_{z,x=c}$ and the prior $c_1 \stackrel{\mathrm{def}}{=} z$. Then we conduct the adjustment by calculating the joint distribution $f(c_1)$ from $f$ and $c_1$ and then taking the marginal distribution $c_0 \stackrel{\mathrm{def}}{=} f(c_1) \downarrow_y$. The resulting $c_0$ is the same as the $c_0$ in the RCT experiment $\varphi_{\mathrm{RCT}}$; that is, the backdoor adjustment $\varphi_{\mathrm{BDA}}$ can compute the causal effect obtained by $\varphi_{\mathrm{RCT}}$.

For this adjustment, we need to check the requirement $pa(z, x) \wedge pos(x :: z)$, that is, $z$ is $x$'s parent in the diagram $G$ and the joint distribution $x :: z$ satisfies the positivity (i.e., it takes each value with a non-zero probability).

Now we formalize the *correctness* of this causal inference method (for any diagram $G$) as the judgment expressing that under the above requirements, the backdoor adjustment computes the same causal effect as the RCT experiment:

$$pa(z, x) \wedge pos(x :: z) \vdash_g \varphi_{\mathrm{RCT}} \leftrightarrow \varphi_{\mathrm{BDA}}. \quad (1)$$

By deriving this judgment in a deductive system called $\mathbf{AX}^{CP}$ (Section 6), we show the correctness of this causal inference method for any diagram (Section 7). We show all proofs of the technical results in this paper's full version [25].

## 3 Language for Data Generation

In this section, we introduce a language for describing data generation.

### 3.1 Constants and Causal Variables

We introduce a set $\mathsf{Const}$ of *constants* to denote probability distributions of data values and a set $\mathsf{dConst} \subseteq \mathsf{Const}$ of *deterministic constants*, each denoting a single data value (strictly speaking, denoting a distribution having a single data value with probability 1).

We introduce a finite set $\mathsf{CVar}$ of *causal variables*. A tuple $\langle x_1, \ldots, x_k \rangle$ of causal variables represents the joint distribution of $k$ variables $x_1, \ldots, x_k$. We denote the set of all non-empty (resp. possibly empty) tuples of variables by $\mathsf{CVar}^+$ (resp. $\mathsf{CVar}^*$). We use the bold font for a *tuple*; e.g., $\boldsymbol{x} = \langle x_1, \ldots, x_k \rangle$. We write $size(\boldsymbol{x})$ for the *dimension* $k$ of a tuple $\boldsymbol{x}$. We assume that the variables in a tuple $\boldsymbol{x}$ are sorted lexicographically.

For disjoint tuples $\boldsymbol{x}$ and $\boldsymbol{y}$, $\boldsymbol{x} :: \boldsymbol{y}$ denotes the *joint distribution* of $\boldsymbol{x}$ and $\boldsymbol{y}$. Formally, '::' is *not* a function symbol, but a meta-operator on $\mathsf{CVar}^*$; $\boldsymbol{x} :: \boldsymbol{y}$ is the tuple obtained by merging $\boldsymbol{x}$ and $\boldsymbol{y}$ and sorting the variables lexicographically.

We use *conditional causal variables* $\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}}$ to denote the conditional distribution of $\boldsymbol{y}$ given $\boldsymbol{z}$ and $\boldsymbol{x} = \boldsymbol{c}$. We write $\mathsf{FVar}$ for the set of all conditional causal variables. For a conditional distribution $\boldsymbol{y}|_{\boldsymbol{x}}$ and a prior distribution $\boldsymbol{x}$, we write $\boldsymbol{y}|_{\boldsymbol{x}}(\boldsymbol{x})$ for the joint distribution $\boldsymbol{x} :: \boldsymbol{y}$.

### 3.2 Terms

We define *terms* to express how data are generated. Let $\mathsf{Fsym}$ be a set of *function symbols* denoting algorithms. We define the set $\mathsf{CTerm}$ of *causal terms* as the terms of depth at most 1; i.e.,

$$u ::= c \mid f(v, \ldots, v)$$

where $c \in \mathsf{Const}$, $f \in \mathsf{Fsym}$, and $v \in \mathsf{CVar} \cup \mathsf{Const}$. For example, $f(c)$ denotes a data generated by an algorithm $f$ with input $c$. We denote the set of variables (resp. the set of constants) occurring in a term $u$ by $\mathsf{fv}(u)$ (resp. $\mathsf{fc}(u)$).

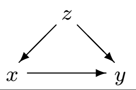
| Data generator $g$ | Causal diagram $G$ given from $g$ |
|---|---|
| $dom(g) = \{x, y, z\}$<br>$f_1(z) \rightarrow_g x$<br>$f_2(z, x) \rightarrow_g y$ |  |

Fig.2: The data generator and causal diagram for Example 1.

We also define the set $\mathsf{Term}$ of *terms* by the BNF:

$$u ::= \boldsymbol{x} \mid c \mid f(u, \dots, u),$$

where $\boldsymbol{x} \in \mathsf{CVar}^+$, $c \in \mathsf{Const}$, and $f \in \mathsf{Fsym} \cup \mathsf{FVar}$. Unlike $\mathsf{CTerm}$, terms in $\mathsf{Term}$ may repeatedly apply functions to describe multiple steps of data generation.

We introduce the special function symbol $\downarrow_{\boldsymbol{x}}$ for marginalization. $\boldsymbol{y}\downarrow_{\boldsymbol{x}}$ denotes the *marginal distribution* of $\boldsymbol{x}$ given a joint distribution $\boldsymbol{y}$; e.g., for a joint distribution $\boldsymbol{x} = \langle x_0, x_1 \rangle$, $\boldsymbol{x}\downarrow_{x_0}$ expresses the marginal distribution $x_0$. We also introduce the special constant $\perp$ for *undefined values.*

### 3.3 Data Generators

To describe how data are generated, we introduce the notion of a *data generator* as a function $g : \mathsf{CVar} \to \mathsf{CTerm} \cup \{\perp\}$ that maps a causal variable $x$ to a causal term representing how the data assigned to $x$ is generated. If $g(y) = u$ for $u \in \mathsf{CTerm}$ and $y \in \mathsf{CVar}$, we write $u \rightarrow_g y$. For instance, the data generator $g$ in Fig. 2 models the situation in Example 1. To express that a variable $x$'s value is generated by an algorithm $f_1$ with an input $z$, the data generator $g$ maps $x$ to $f_1(z)$, i.e., $f_1(z) \rightarrow_g x$. Since the causal term $f_1(z)$'s depth is at most 1, $z$ represents the *direct cause* of $x$. We denote the set of all variables $x$ satisfying $g(x) \neq \perp$ by $dom(g)$, and the range of $g$ by $range(g)$.

We assume the following *at-most-once* condition: Each function symbol and constant can be used at most once in a single data generator. This ensures that different sampling uses different randomness and is denoted by different symbols.

We say that a data generator $g$ is *finite* if $dom(g)$ is a finite set. We say that a data generator $g$ is *closed* if no undefined variable occurs in the terms that $g$ assigns to variables, namely, $\mathsf{fv}(range(g)) \subseteq dom(g)$.

We write $x \prec_g y$ iff $y$'s value depends on $x$'s, i.e., there are variables $z_1, \dots, z_i$ $(i \geq 2)$ such that $z_1 = x$, $z_i = y$, and $z_j \in \mathsf{fv}(g(z_{j+1}))$ for $1 \leq j \leq i - 1$. A data generator $g$ is *acyclic* if $\prec_g$ is a strict partial order over $dom(g)$. Then we can avoid the cyclic definitions of $g$. E.g., the data generator $g_1$ defined by $f(z) \rightarrow_{g_1} x$ and $f(c) \rightarrow_{g_1} z$ is acyclic, whereas $g_2$ by $f(z) \rightarrow_{g_2} x$ and $f(x) \rightarrow_{g_2} z$ is cyclic.

## 4 Kripke Model for Statistical Causality

In this section, we introduce a Kripke model for statistical causality.

We write $\mathcal{O}$ for the set of all data values we deal with, such as the Boolean values, integers, real numbers, and lists of data values. We write $\perp$ for the undefined value. For a set $S$, we denote the set of all probability distributions over $S$ by $\mathbb{D}S$. For a probability distribution $m \in \mathbb{D}S$, we write $supp(m)$ for the set of $m$'s non-zero probability elements.

### 4.1 Causal Diagrams

To model causal relations corresponding to a given data generator $g$, we consider a *causal diagram* $G = (U, V, E)$ [31] where $U \cup V$ is the set of all nodes and $E$ is the set of all edges such that:

- $U \stackrel{\text{def}}{=} \mathsf{fc}(range(g)) \subseteq \mathsf{Const}$ is a set of symbols called *exogenous variables* that denote distributions of data;
- $V \stackrel{\text{def}}{=} dom(g) \subseteq \mathsf{CVar}$ is a set of symbols called *endogenous variables* that may depend on other variables;
- $E \stackrel{\text{def}}{=} \{x \to y \in V \times V \mid x \in \mathsf{fv}(g(y))\} \cup \{c \to y \in$

$U \times V \mid c \in \mathsf{fc}(g(y))\}$ is the set of all *structural equations*, i.e., directed edges (arrows) denoting the direct causal relations between variables defined by the data generator $g$.

For instance, in Fig. 2, Example 1 is modeled as the causal diagram $G$.

Since a causal term's depth is at most 1, $g$ specifies all information for defining $G$. By $g$'s acyclicity, $G$ is a directed acyclic graph (DAG) (See Proposition 4 in the full version [25] for details).

### 4.2 Pre-/Post-Intervention Distributions

For a causal diagram $G = (U, V, E)$ and a tuple $\boldsymbol{y} \subseteq V$, we write $P_G(\boldsymbol{y})$ for the joint distribution of $\boldsymbol{y}$ over $\mathcal{O}^{size(\boldsymbol{y})}$ generated according to $G$. As shown in the standard textbooks (e.g., [31]), $P_G(V)$ is factorized into conditional distributions according to $G$ as follows:

$$P_G(V) \stackrel{\text{def}}{=} \prod_{y_i \in V} P_G(y_i \mid pa_G(y_i)), \qquad (2)$$

where $pa_G(y_i)$ is the set of parent variables of $y_i$ in $G$. For example, in Fig. 2, for $V = \{x, y, z\}$, $P_G(V) = P_G(y \mid x, z)\, P_G(x \mid z)\, P_G(z)$.

For tuples $\boldsymbol{x} \subseteq V$ and $\boldsymbol{o} \subseteq \mathcal{O}$ with $size(\boldsymbol{x}) = size(\boldsymbol{o})$, the *post-intervention distribution* $P_G(V \mid do(\boldsymbol{x}{=}\boldsymbol{o}))$ is the joint distribution of $V$ after $\boldsymbol{x}$ is assigned $\boldsymbol{o}$ and all the variables dependent on $\boldsymbol{x}$ in $G$ are updated by $\boldsymbol{x} := \boldsymbol{o}$ as follows:

$$P_G(V \mid do(\boldsymbol{x} = \boldsymbol{o})) \stackrel{\text{def}}{=} \begin{cases} \prod_{y_i \in V \setminus \boldsymbol{x}} P_G(y_i \mid pa_G(y_i)) & \\ \quad \text{for values of } V \text{ consistent with } \boldsymbol{x} = \boldsymbol{o} & \\ 0 \quad \text{otherwise.} & \end{cases}$$

For instance, in Fig. 2, $P_G(y, z | do(x = o)) = P_G(y | x = o, z)\, P_G(z)$ for any $o \in \mathcal{O}$.

### 4.3 Possible Worlds

We introduce the notion of a *possible world* to define the probability distribution of causal variables from a data generator. Formally, a possible world is a tuple $(g, \xi, m)$ of (i) a finite and acyclic data generator $g : \mathsf{CVar} \to \mathsf{CTerm} \cup \{\bot\}$, (ii) an interpretation $\xi$ that maps a function symbol in $\mathsf{Fsym}$ with arity $k \geq 0$ to a function from $\mathcal{O}^k$ to $\mathbb{D}\mathcal{O}$, and (iii) a memory $m$ that maps a tuple of variables to a joint distribution of data values, which is determined by $g$ and $\xi$. We denote these components of a world $w$ by $g_w$, $\xi_w$, and $m_w$, and the set of all defined variables in $w$ by $\mathsf{Var}(w) = dom(m_w)$.

The interpretation $\xi$ can be constructed using a probability distribution $I$ over an index set $\mathcal{I}$ and a family $\{\xi^r\}_{r \in \mathcal{I}}$ of interpretations each mapping a function symbol $f$ with arity $k \geq 0$ to a deterministic function $\xi^r(f)$ from $\mathcal{O}^k$ to $\mathcal{O}$. Then $\xi(f)$ maps data values $\boldsymbol{o}$ to the probability distribution over $\mathcal{O}$ obtained by randomly drawing an index $r$ from $I$ and then computing $\xi^r(f)(\boldsymbol{o})$.

If $k = 0$, $f$ is a constant and $\xi^r(f) \in \mathcal{O}$, hence $\xi(f) \in \mathbb{D}\mathcal{O}$ is a distribution of data values. For the undefined constant, we assume $\xi^r(\bot) = \bot$.

### 4.4 Interpretation of Terms

Terms are interpreted in a possible world $w = (\xi, g, m)$ as follows. First, for each index $r \in \mathcal{I}$, we define the *interpretation* $[\![\_]\!]^r_{\xi,g}$ that maps a tuple of $k$ terms to $k$ data values in $\mathcal{O}$ or $\bot$ by:

$$\begin{aligned} [\![\boldsymbol{x}]\!]^r_{\xi,g} &= [\![g(\boldsymbol{x})]\!]^r_{\xi,g} \\ [\![c]\!]^r_{\xi,g} &= \xi^r(c) \\ [\![\langle u_1, \ldots, u_k \rangle]\!]^r_{\xi,g} &= ([\![u_1]\!]^r_{\xi,g}, \ldots, [\![u_k]\!]^r_{\xi,g}) \\ [\![f(u_1, \ldots, u_k)]\!]^r_{\xi,g} &= \xi^r(f)([\![\langle u_1, \ldots, u_k \rangle]\!]^r_{\xi,g}). \end{aligned}$$

For instance, in Fig. 2, we have $[\![x]\!]^r_{\xi,g} = [\![g(x)]\!]^r_{\xi,g} = [\![f_1(z)]\!]^r_{\xi,g} = \xi^r(f_1)([\![z]\!]^r_{\xi,g})$, where the interpretation of $z$ does not depend on that of $x$ due to $g$'s acyclicity. We define the probability distribution $[\![u]\!]_w$ over $\mathcal{O}$ by randomly drawing $r$ and then computing $[\![u]\!]^r_{\xi,g}$. Similarly, we define $[\![\langle u_1, \ldots, u_k \rangle]\!]_w$ via $[\![\langle u_1, \ldots, u_k \rangle]\!]^r_{\xi,g}$.

We remark that the interpretation $[\![\_]\!]_w$ defines the joint distribution $P_{G_w}$ of all variables in the causal diagram $G_w$; e.g., $[\![\boldsymbol{y}|_{\boldsymbol{z}}]\!]_w = P_{G_w}(\boldsymbol{y} \mid \boldsymbol{z})$ (See Proposition 5 in the full version [25] for details).

A function symbol $f$ is interpreted as the function $\xi(f)$ that maps data values in $\mathcal{O}$ to the distribution over $\mathcal{O}$. We define the memory $m$ by $m(\boldsymbol{x}) = [\![\boldsymbol{x}]\!]_w$ for all $\boldsymbol{x} \in \mathsf{CVar}^+$. Notice that $[\![\_]\!]_w$ is defined using $g$ and $\xi$ without using $m$.

We expand the interpretation $[\![\_]\!]_w$ to a conditional causal variable $\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}} \in \mathsf{FVar}$ to interpret it as a function that maps a value $\boldsymbol{c}'$ of $\boldsymbol{z}$ to the distribution $[\![(\boldsymbol{x} :: \boldsymbol{y} :: \boldsymbol{z})|_{\boldsymbol{z}=\boldsymbol{c}',\boldsymbol{x}=\boldsymbol{c}}]\!]_w$. We then have $[\![\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}}(\boldsymbol{z}|_{\boldsymbol{x}=\boldsymbol{c}})]\!]_w = [\![\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}}]\!]_w([\![\boldsymbol{z}|_{\boldsymbol{x}=\boldsymbol{c}}]\!]_w)$.

For the sake of reasoning in Section 6, for each data generator $g$, $\boldsymbol{x} \in \mathsf{CVar}^+$, and $\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}} \in \mathsf{FVar}$, we introduce a constant $c^{(g,\boldsymbol{x})}$ and a function symbol $f^{(g,\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}})}$. For brevity, we often omit the superscripts of these symbols.

### 4.5 Eager/Lazy Interventions

We introduce two forms of *interventions* and their corresponding *intervened worlds*. Intuitively, in a causal diagram, an *eager intervention* $\lceil c/x \rceil$ expresses the removal of all arrows pointing *to* a variable $x$ by replacing $x$'s value with $c$.

In contrast, a *lazy intervention* $\lfloor c/x \rfloor$ expresses the removal of all arrows emerging *from* $x$, which does not change the value of $x$ itself but affects the values of the variables dependent on $x$, computed using $[\![c]\!]$ (instead of $[\![x]\!]$) as the value of $x$.

For instance, Fig. 3 shows how two interventions $\lceil c/x \rceil$ and $\lfloor c/x \rfloor$ change the data generator and the causal diagram in a world $w$ that models Example 1.

For a world $w$ and a $c \in \mathsf{dConst}$, we define an *eagerly intervened world* $w\lceil c/x \rceil$ as the world where $[\![c]\!]_w$ is assigned to $x$ and is used to compute the other variables dependent on $x$. Formally, $w\lceil c/x \rceil$ is defined by $\xi_{w\lceil c/x \rceil} = \xi_w$, $g_{w\lceil c/x \rceil}(y) = c$ if $y = x$, and $g_{w\lceil c/x \rceil}(y) = g_w(y)$ if $y \neq x$. For instance, in Fig. 3, in the world $w\lceil c/x \rceil$, we use the value of $c$ to compute $[\![x]\!]_{w\lceil c/x \rceil} = \xi_w(c)$ and $[\![y]\!]_{w\lceil c/x \rceil} = [\![f_2(z, x)]\!]_{w\lceil c/x \rceil} = [\![f_2(z, c)]\!]_w$.

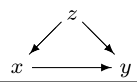
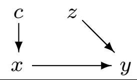
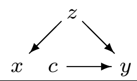
| World | Data generator | Causal diagram |
|---|---|---|
| $w$ | $f_1(z) \to x$; $f_2(z, x) \to y$ |  |
| $w\lceil c/x \rceil$ | $c \to x$; $f_2(z, x) \to y$ |  |
| $w\lfloor c/x \rfloor$ | $f_1(z) \to x$; $f_2(z, c) \to y$ |  |

Fig.3: Eager/lazy interventions.

Then the interpretation $[\![\_]\!]_{w\lceil c/x \rceil}$ defines the joint distribution of all variables in the causal diagram $G_w$ after the intervention $\boldsymbol{x} := [\![\boldsymbol{c}]\!]_w$; e.g., $[\![\boldsymbol{y}|_{\boldsymbol{z}}]\!]_{w\lceil \boldsymbol{c}/\boldsymbol{x} \rceil} = P_{G_w}(\boldsymbol{y} \mid do(\boldsymbol{x} = [\![\boldsymbol{c}]\!]_w), \boldsymbol{z})$ (See Proposition 5 in the full version [25] for details).

We next define a *lazily intervened world* $w\lfloor c/x \rfloor$ as the world where $x$'s value is unchanged but the other variables dependent on $x$ are computed using $[\![c]\!]_w$ instead of $[\![x]\!]_w$. Formally, $w\lfloor c/x \rfloor$ is defined by $\xi_{w\lfloor c/x \rfloor} = \xi_w$, $g_{w\lfloor c/x \rfloor}(y) = x$ if $y = x$, and $g_{w\lfloor c/x \rfloor}(y) = g_w(y)[x \mapsto c]$ if $y \neq x$. E.g., in Fig. 3, $[\![x]\!]_{w\lfloor c/x \rfloor} = [\![f_1(z)]\!]_w$.

For $\boldsymbol{x} = \langle x_1, \ldots, x_k \rangle$ and $\boldsymbol{c} = \langle c_1, \ldots, c_k \rangle$, we define $\lceil \boldsymbol{c}/\boldsymbol{x} \rceil$ from the simultaneous replacement $g_{w\lceil c_1/x_1, \ldots, c_k/x_k \rceil}$. We also define $\lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor$ analogously.

### 4.6 Kripke Model.

Let $\mathsf{Psym}$ be a set of predicate symbols. For a variable tuple $\boldsymbol{x}$ and a deterministic constant tuple $\boldsymbol{c}$, we introduce an *intervention relation* $w\mathcal{R}_{\lceil \boldsymbol{c}/\boldsymbol{x} \rceil}w'$ that expresses a transition from a world $w$ to another $w'$ by the intervention $\lceil \boldsymbol{c}/\boldsymbol{x} \rceil$; namely, $\mathcal{R}_{\lceil \boldsymbol{c}/\boldsymbol{x} \rceil} = \{(w, w') \in \mathcal{W} \times \mathcal{W} \mid w' = w\lceil \boldsymbol{c}/\boldsymbol{x} \rceil\}$.

Then we define a *Kripke model for statistical causality* as a tuple $\mathfrak{M} = (\mathcal{W}, (\mathcal{R}_{\lceil \boldsymbol{c}/\boldsymbol{x} \rceil})_{\boldsymbol{x} \in \mathsf{CVar}^+, \boldsymbol{c} \in \mathsf{dConst}^+}, \mathcal{V})$ consisting of:

(1) a set $\mathcal{W}$ of all possible worlds over the set $\mathsf{CVar}$ of causal variables;

(2) for each $\boldsymbol{x} \in \mathsf{CVar}^+$ and $\boldsymbol{c} \in \mathsf{dConst}^+$, an *intervention relation* $\mathcal{R}_{\lceil \boldsymbol{c}/\boldsymbol{x} \rceil}$;

(3) a valuation $\mathcal{V}$ that maps a $k$-ary predicate symbol $\eta \in \mathsf{Psym}$ to a set $\mathcal{V}(\eta)$ of $k$-tuples of distributions.

Notice that different worlds $w$ and $w'$ in $\mathcal{W}$ may have different data generators $g_w$ and $g_{w'}$ corresponding to different causal diagrams; that is, $\mathcal{W}$ specifies all possible causal diagrams. Furthermore, different worlds $w$ and $w'$ may also have different interpretations $\xi_w$ and $\xi_{w'}$ of function symbols if we do not have the knowledge of functions [23].

## 5 Statistical Causality Language

### 5.1 Predicates and Causal Predicates

Classical predicates in Psym describe *statistical correlation* among the distributions of variables, and are interpreted using a valuation $\mathcal{V}$. For example, $pos(x)$ expresses that $x$ takes each value in the domain $\mathcal{O}$ with a non-zero probability. However, predicates cannot express the *statistical causality* among variables, whose interpretation relies on a causal diagram. Thus, we introduce a set CPsym of *causal predicates* (e.g., *dsep*, *nanc*, *allnanc*) and interpret them using a data generator $g$ instead of a valuation $\mathcal{V}$.

### 5.2 Syntax and Semantics of StaCL

We define the set Fml of *formulas*: For $\eta \in \mathsf{Psym}$, $\chi \in \mathsf{CPsym}$, $\boldsymbol{x} \in \mathsf{Var}^+$, $\boldsymbol{u} \in \mathsf{Term}^+$, $\boldsymbol{c} \in \mathsf{Const}^+$, and $f \in \mathsf{Fsym} \cup \mathsf{FVar}$,

$$\varphi ::= \eta(\boldsymbol{x}, \dots, \boldsymbol{x}) \mid \chi(\boldsymbol{x}, \dots, \boldsymbol{x}) \mid \boldsymbol{u} = \boldsymbol{u} \mid f = f \mid \mathtt{true} \mid \neg\varphi \mid \varphi \wedge \varphi \mid \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi \mid \lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \varphi.$$

Intuitively, $\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi$ (resp. $\lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \varphi$) expresses that $\varphi$ is satisfied in the eager (resp. lazy) intervened world. We assume that each variable appears at most once in $\boldsymbol{x}$ in $\lceil \boldsymbol{c}/\boldsymbol{x} \rceil$ and $\lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor$. We use syntax sugar `false`, $\vee$, $\rightarrow$, and $\leftrightarrow$ as usual. Note that the formulas have no quantifiers over variables.

We interpret a formula in a world $w$ in a Kripke model $\mathfrak{M}$ by:

$$\begin{aligned}
\mathfrak{M}, w \models \eta(\boldsymbol{x}_1, \dots, \boldsymbol{x}_k) &\text{ iff } (\llbracket x_1 \rrbracket_w, \dots, \llbracket x_k \rrbracket_w) \in \mathcal{V}(\eta) \\
\mathfrak{M}, w \models \boldsymbol{u} = \boldsymbol{u}' &\text{ iff } \llbracket \boldsymbol{u} \rrbracket_w = \llbracket \boldsymbol{u}' \rrbracket_w \\
\mathfrak{M}, w \models f = f' &\text{ iff } \llbracket f \rrbracket_w = \llbracket f' \rrbracket_w \\
\mathfrak{M}, w \models \neg\varphi &\text{ iff } \mathfrak{M}, w \not\models \varphi \\
\mathfrak{M}, w \models \varphi \wedge \varphi' &\text{ iff } \mathfrak{M}, w \models \varphi \text{ and } \mathfrak{M}, w \models \varphi' \\
\mathfrak{M}, w \models \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi &\text{ iff } \mathfrak{M}, w\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \models \varphi \\
\mathfrak{M}, w \models \lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \varphi &\text{ iff } \mathfrak{M}, w\lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \models \varphi,
\end{aligned}$$

where $w\lceil \boldsymbol{c}/\boldsymbol{x} \rceil$ and $w\lfloor \boldsymbol{u}/\boldsymbol{x} \rfloor$ are intervened worlds and the interpretation of atomic formulas with causal predicates $\chi$ is given below. For brevity, we often omit $\mathfrak{M}$.

Note that $\eta(x_1, \dots, x_k)$ represents a property of $k$ independent distributions $\llbracket x_1 \rrbracket_w, \dots, \llbracket x_k \rrbracket_w$, where the randomness $r_i$ in each $\llbracket x_i \rrbracket_w^{r_i}$ is chosen independently. In contrast, $\eta(\langle x_1, \dots, x_k \rangle)$ expresses a property of a single joint distribution, since the same $r$ is used in all of $\llbracket x_1 \rrbracket_w^r$, …, $\llbracket x_k \rrbracket_w^r$.

Atomic formulas with causal predicates $\chi$ are interpreted using a causal diagram $G_w$ corresponding to $g_w$. Let $\mathtt{ANC}(\boldsymbol{y})$ is the set of all ancestors of $\boldsymbol{y}$ in $G_w$, and $\mathtt{PA}(\boldsymbol{y})$ be the set of all parent variables of $\boldsymbol{y}$ in $G_w$. Then:

$$\begin{aligned}
w \models dsep(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) &\text{ iff } \boldsymbol{x} \text{ and } \boldsymbol{y} \text{ are } d\text{-separated by } \boldsymbol{z} \text{ in } G_w \\
w \models nanc(\boldsymbol{x}, \boldsymbol{y}) &\text{ iff } \boldsymbol{x} \cap \mathtt{ANC}(\boldsymbol{y}) = \emptyset \text{ and } \boldsymbol{x} \cap \boldsymbol{y} = \emptyset \\
w \models allnanc(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) &\text{ iff } \boldsymbol{x} = \boldsymbol{y} \setminus \mathtt{ANC}(\boldsymbol{z}) \\
w \models pa(\boldsymbol{x}, \boldsymbol{y}) &\text{ iff } \boldsymbol{x} = \mathtt{PA}(\boldsymbol{y}) \text{ and } \boldsymbol{x} \cap \boldsymbol{y} = \emptyset,
\end{aligned}$$

where the $d$-separation †1 of $\boldsymbol{x}$ and $\boldsymbol{y}$ by $\boldsymbol{z}$ [36] is a sufficient condition for the conditional independence of $\boldsymbol{x}$ and $\boldsymbol{y}$ given $\boldsymbol{z}$ (See Appendix A in the full version [25]).

### 5.3 Formalization of Causal Effect.

Conventionally, the conditional probability of $\boldsymbol{y}$ given $\boldsymbol{z} = \boldsymbol{o_2}$ after an intervention $\boldsymbol{x} = \boldsymbol{o_1}$ is

†1 An undirected path in a causal diagram $G_w$ is said to be *d-separated* by $\boldsymbol{z}$ if it has either (a) a chain $v' \rightarrow v \rightarrow v''$ s.t. $v \in \boldsymbol{z}$, (b) a fork $v' \leftarrow v \rightarrow v''$ s.t. $v \in \boldsymbol{z}$, or (c) a collider $v' \rightarrow v \leftarrow v''$ s.t. $v \notin \boldsymbol{z} \cup \mathtt{ANC}(\boldsymbol{z})$. $\boldsymbol{x}$ and $\boldsymbol{y}$ are said to be *d-separated* by $\boldsymbol{z}$ if all undirected paths between variables in $\boldsymbol{x}$ and in $\boldsymbol{z}$ are $d$-separated by $\boldsymbol{z}$.

expressed using the *do*-operator by $P(\boldsymbol{y} \mid do(\boldsymbol{x} = \boldsymbol{o_1}), \boldsymbol{z} = \boldsymbol{o_2})$. This causal effect can be expressed using StaCL:

**Proposition 1 (Causal effect)** *Let $w$ be a world, $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathsf{Var}(w)^+$ be disjoint, $\boldsymbol{c} \in \mathsf{dConst}^+$, $\boldsymbol{c}' \in \mathsf{Const}^+$, and $f \in \mathsf{Fsym}$. Then:*

*(i) $w \models \lceil \boldsymbol{c}/\boldsymbol{x} \rceil (\boldsymbol{c}' = \boldsymbol{y})$ iff there is a distribution $P_{G_w}$ that is factorized according to $G_w$ and satisfies $P_{G_w}(\boldsymbol{y} \mid do(\boldsymbol{x} = \boldsymbol{c})) = [\![\boldsymbol{c}']\!]_w$.*

*(ii) $w \models \lceil \boldsymbol{c}/\boldsymbol{x} \rceil (f = \boldsymbol{y}|_{\boldsymbol{z}})$ iff there is a distribution $P_{G_w}$ that is factorized according to $G_w$ and satisfies $P_{G_w}(\boldsymbol{y} \mid do(\boldsymbol{x} = \boldsymbol{c}), \boldsymbol{z}) = [\![f]\!]_w$.*

If $\boldsymbol{x}$ and $\boldsymbol{y}$ are $d$-separated by $\boldsymbol{z}$, they are conditionally independent given $\boldsymbol{z}$ [36] (but not vice versa). StaCL can express this by $\models_g (dsep(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \wedge pos(\boldsymbol{z}) \rightarrow \boldsymbol{y}|_{\boldsymbol{z}, \boldsymbol{x}=\boldsymbol{c}} = \boldsymbol{y}|_{\boldsymbol{z}}$, where $pos(\boldsymbol{z})$ means that $\boldsymbol{z}$ takes each value with a positive probability, and $\models_g \varphi$ is defined as $w \models_g \varphi$ for all world $w$ having the data generator $g$. Furthermore, if $[\![\boldsymbol{x}]\!]_w$ and $[\![\boldsymbol{y}]\!]_w$ are conditionally independent given $[\![\boldsymbol{z}]\!]_w$ for any world $w$ with the data generator $g_w$, then they are $d$-separated by $\boldsymbol{z}$: $\models_g \boldsymbol{y}|_{\boldsymbol{z}, \boldsymbol{x}=\boldsymbol{c}} = \boldsymbol{y}|_{\boldsymbol{z}} \wedge pos(\boldsymbol{z})$ implies $\models_g dsep(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ (See Proposition 15 in the full version [25] for details).

## 6 Axioms for StaCL

We present a sound deductive system for StaCL in the Hilbert style. Our system consists of axioms and rules for the judgments of the form $\Gamma \vdash_g \varphi$.

The deductive system is stratified into two groups. The system **AX**, determined by the axioms in Figs. 4 and 5, concerns the derivation of $\Gamma \vdash_g \varphi$ that does not involve causal predicates (e.g., *pa*, *nanc*, *dsep*). The system $\mathbf{AX}^{CP}$, determined by the axioms in Fig. 6, concerns the derivation of a formula $\varphi$ possibly equipped with causal predicates in a judgment $\Gamma \vdash_g \varphi$.

In these systems, we deal only with the reasoning that is independent of a causal diagram. Indeed, in Section 7, we will present examples of reasoning using the deductive system $\mathbf{AX}^{CP}$ that do not refer to a specific causal diagram.

### 6.1 Axioms of AX

Fig. 4 shows the axioms of the deductive system **AX**, where we omitted the axioms for propositional logic and equations (PT for the propositional tautologies, MP for the modus ponens, Eq1 for the reflexivity, and Eq2 for the substitutions for formulas). $\mathrm{Eq_C}$ and $\mathrm{Eq_F}$ represent the definitions of constants and function symbols corresponding to causal variables. PD describes the relationships among the prior distribution $\boldsymbol{x}$, the conditional distribution $\boldsymbol{y}|_{\boldsymbol{x}}$ of $\boldsymbol{y}$ given $\boldsymbol{x}$, and the joint distribution $\boldsymbol{x} :: \boldsymbol{y}$. MPD represents the computation $\downarrow_{\boldsymbol{x_2}}$ of the marginal distribution $\boldsymbol{x_2}$ from a joint distribution $\boldsymbol{x_1}$.

The axioms named with the subscript EI deal with eager intervention. Remarkably, $\mathrm{DG_{EI}}$ reduces the derivation of $\vdash_g \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi$, which involves an intervention modality $\lceil \boldsymbol{c}/\boldsymbol{x} \rceil$, to the derivation of $\vdash_{g \lceil \boldsymbol{c}/\boldsymbol{x} \rceil} \varphi$, which does not involve the modality under the modified data generator $g\lceil \boldsymbol{c}/\boldsymbol{x} \rceil$. The axioms $\mathrm{Distr_{EI}}^{\neg}$ and $\mathrm{Distr_{EI}}^{\wedge}$ allow for pushing intervention operators outside logical connectives.

The axioms with the subscript LI deal with lazy intervention; they are analogous to the corresponding EI-rules. The axioms with the subscript EILI describe when an eager intervention can be exchanged with a lazy intervention.

### 6.2 Axioms of $\mathbf{AX}^{CP}$

Fig. 6 shows the axioms for $\mathbf{AX}^{CP}$. DsepCI represents that $d$-separation implies conditional independence. DsepSm, DsepDc, DsepWu, and DsepCn are the *semi-graphoid* axioms [36], characterizing the $d$-separation. However, these well-known axioms are not sufficient to derive the relationships between $d$-separation and interven-

**Axioms for probability distributions**

| | |
|---|---|
| $\textsc{Eq}_{\mathrm{C}}$ | $\vdash_g c^{(g,\boldsymbol{x})} = \boldsymbol{x}$ |
| $\textsc{Eq}_{\mathrm{F}}$ | $\vdash_g f^{(g,\boldsymbol{y}\|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}})} = \boldsymbol{y}\|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}}$ |
| PD | $\vdash_g (pos(\boldsymbol{x}) \wedge c_0 = \boldsymbol{x} \wedge f = \boldsymbol{y}\|_{\boldsymbol{x}} \wedge c_1 = \boldsymbol{x} :: \boldsymbol{y}) \rightarrow c_1 = f(c_0)$ |
| MPD | $\vdash_g \boldsymbol{x_1} \downarrow_{\boldsymbol{x_2}} = \boldsymbol{x_2}$ if $\boldsymbol{x_2} \subseteq \boldsymbol{x_1}$ |

Fig.4: The axioms of **AX** for probability distributions, where $\boldsymbol{x}, \boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{y} \in \mathsf{CVar}^+$ are disjoint, $c_0, c_1, c^{(g,\boldsymbol{x})} \in \mathsf{Const}$, $f, f^{(g,\boldsymbol{y}|_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}})} \in \mathsf{Fsym}$.

**Axioms for eager interventions**

| | |
|---|---|
| $\textsc{DG}_{\mathrm{EI}}$ | $\vdash_g \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi$ iff $\vdash_{g\lceil \boldsymbol{c}/\boldsymbol{x} \rceil} \varphi$ |
| $\textsc{Effect}_{\mathrm{EI}}$ | $\vdash_g \lceil \boldsymbol{c}/\boldsymbol{x} \rceil (\boldsymbol{x} = \boldsymbol{c})$ |
| $\textsc{Eq}_{\mathrm{EI}}$ | $\vdash_g \boldsymbol{u_1} = \boldsymbol{u_2} \leftrightarrow \lceil \boldsymbol{c}/\boldsymbol{x} \rceil (\boldsymbol{u_1} = \boldsymbol{u_2})$ if $\mathsf{fv}(\boldsymbol{u_1}) = \mathsf{fv}(\boldsymbol{u_2}) = \emptyset$ |
| $\textsc{Split}_{\mathrm{EI}}$ | $\vdash_g \lceil \boldsymbol{c_1}/\boldsymbol{x_1}, \boldsymbol{c_2}/\boldsymbol{x_2} \rceil \varphi \rightarrow \lceil \boldsymbol{c_1}/\boldsymbol{x_1} \rceil \lceil \boldsymbol{c_2}/\boldsymbol{x_2} \rceil \varphi$ |
| $\textsc{Simul}_{\mathrm{EI}}$ | $\vdash_g \lceil \boldsymbol{c_1}/\boldsymbol{x_1} \rceil \lceil \boldsymbol{c_2}/\boldsymbol{x_2} \rceil \varphi \rightarrow \lceil \boldsymbol{c'_1}/\boldsymbol{x'_1}, \boldsymbol{c_2}/\boldsymbol{x_2} \rceil \varphi$ if $\boldsymbol{x'_1} = \boldsymbol{x_1} \backslash \boldsymbol{x_2}$, $\boldsymbol{c'_1} = \boldsymbol{c_1} \backslash \boldsymbol{c_2}$ |
| $\textsc{Rpt}_{\mathrm{EI}}$ | $\vdash_g \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi \rightarrow \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi$ |
| $\textsc{Cmp}_{\mathrm{EI}}$ | $\vdash_g \big(\lceil \boldsymbol{c_1}/\boldsymbol{x_1} \rceil (\boldsymbol{x_2} = \boldsymbol{c_2}) \wedge \lceil \boldsymbol{c_1}/\boldsymbol{x_1} \rceil (\boldsymbol{x_3} = \boldsymbol{u})\big) \rightarrow \lceil \boldsymbol{c_1}/\boldsymbol{x_1}, \boldsymbol{c_2}/\boldsymbol{x_2} \rceil (\boldsymbol{x_3} = \boldsymbol{u})$ |
| $\textsc{Distr}_{\mathrm{EI}}{}^{\neg}$ | $\vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \neg\varphi) \leftrightarrow (\neg \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi)$ |
| $\textsc{Distr}_{\mathrm{EI}}{}^{\wedge}$ | $\vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil (\varphi_1 \wedge \varphi_2)) \leftrightarrow (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi_1 \wedge \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \varphi_2)$ |

**Axioms for lazy interventions**

| | |
|---|---|
| $\textsc{Cond}_{\mathrm{LI}}$ | $\vdash_g (f = \boldsymbol{y}\|_{\boldsymbol{x}=\boldsymbol{c}}) \leftrightarrow \lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor (f = \boldsymbol{y}\|_{\boldsymbol{x}=\boldsymbol{c}})$ |

Other axioms are analogous to eager interventions except for $\textsc{Effect}_{\mathrm{EI}}$.

**Axioms for the exchanges of eager and lazy interventions**

| | |
|---|---|
| $\textsc{Expd}_{\mathrm{EILI}}$ | $\vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \boldsymbol{c'} = \boldsymbol{y}) \leftrightarrow (\lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \boldsymbol{c'} = \boldsymbol{y})$ |
| $\textsc{Excd}_{\mathrm{EILI}}$ | $\vdash_g pos(\boldsymbol{z}) \rightarrow \big((\lceil \boldsymbol{c}/\boldsymbol{x} \rceil f = \boldsymbol{y}\|_{\boldsymbol{z}}) \leftrightarrow (\lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor f = \boldsymbol{y}\|_{\boldsymbol{z}})\big)$ |

Fig.5: The axioms of **AX**, where $\boldsymbol{x}, \boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{y}, \boldsymbol{z} \in \mathsf{CVar}^+$ are disjoint, $f \in \mathsf{Fsym}$, $\boldsymbol{c}, \boldsymbol{c_1}, \boldsymbol{c_2} \in \mathsf{dConst}^+$, $\boldsymbol{c'} \in \mathsf{Const}^+$, $\boldsymbol{u}, \boldsymbol{u_1}, \boldsymbol{u_2} \in \mathsf{Term}^+$, and $\varphi, \varphi_1, \varphi_2 \in \mathsf{Fml}$.

tions. Therefore, we introduce two axioms $\textsc{Dsep}_{\mathrm{EI}}$ and $\textsc{Dsep}_{\mathrm{LI}}$ in Fig. 6 for the *d*-separation before/after interventions, and four axioms to reason about the relationships between the causal predicate *nanc* and the interventions/*d*-separation (named $\textsc{Nanc}_{\{1,2,3,4\}}$ in Fig. 6). By $\textsc{AllNanc}$, $\textsc{PaNanc}$, and $\textsc{PaDsep}$, we transform the formulas using *allnanc* and *pa* into those with *nanc* or *dsep*.

### 6.3 Properties of Axiomatization

For a data generator $g$, a set $\Gamma \stackrel{\text{def}}{=} \{\psi_1, \ldots, \psi_n\}$ of formulas, and a formula $\varphi$, we write $\Gamma \vdash_g \varphi$ if there is a derivation of $\vdash_g (\psi_1 \wedge \cdots \wedge \psi_n) \rightarrow \varphi$ using axioms of **AX** or $\mathbf{AX}^{CP}$. We write $\Gamma \models_g \varphi$ if for all model $\mathfrak{M}$ and all world $w$ having the data generator $g$, $\mathfrak{M}, w \models \varphi$. Then we obtain the *soundness* of **AX** and $\mathbf{AX}^{CP}$.

**Theorem 1 (Soundness)** *Let $g$ be a finite, closed, and acyclic data generator. $\Gamma \subseteq \mathsf{Fml}$, and $\varphi \in \mathsf{Fml}$. If $\Gamma \vdash_g \varphi$ then $\Gamma \models_g \varphi$.*

We show the proof in Appendices B and C in the full version [25]. As shown in Section 7, $\mathbf{AX}^{CP}$ is expressive enough to derive the rules of Pearl's do-calculus [30]; it can reason about all causal effects identifiable by the do-calculus (without referring to a specific causal diagram). Furthermore, **AX** includes/derives the axioms used in the previous work [1] that are complete w.r.t. a different semantics without dealing with probability distributions. We leave investigating whether **AX** is complete w.r.t. our Kripke model for future work. We also remark that $\mathbf{AX}^{CP}$ has axioms corresponding to the composition and effectiveness axioms introduced by Galles and Pearl [9].

## 7 Reasoning About Statistical Causality

### 7.1 Deriving the Rules of the Do-Calculus

Using StaCL, we express the *do-calculus*'s rules [30], which are sufficient to compute all identifiable causal effects from observable quantities [16] [33]. Let $\mathsf{fv}(\varphi)$ be the set of all variables occurring in a formula $\varphi$, and $\mathsf{cdv}(\varphi)$ be the set of all *conditioning variables* in $\varphi$.

**Proposition 2 (Do-calculus rules)** *Let* $\boldsymbol{v}, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathit{CVar}^{+}$ *be disjoint,* $\boldsymbol{x_1}, \boldsymbol{x_2} \in \mathit{CVar}^{+}$*, and* $\boldsymbol{c_0}, \boldsymbol{c_1}, \boldsymbol{c_2} \in \mathit{dConst}^{+}$*. Let* $S = \mathit{cdv}(\varphi_0) \cup \mathit{cdv}(\varphi_1)$*.*

1. Do1. *Introduction/elimination of conditioning:*
$$\vdash_g \lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil (\mathit{dsep}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \wedge \textstyle\bigwedge_{\boldsymbol{s} \in S} \mathit{pos}(\boldsymbol{s})) \rightarrow ((\lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil \varphi_0) \leftrightarrow \lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil \varphi_1)$$
*where* $\varphi_1$ *is obtained by replacing some occurrences of* $\boldsymbol{y}|_{\boldsymbol{z}}$ *in* $\varphi_0$ *with* $\boldsymbol{y}|_{\boldsymbol{z}, \boldsymbol{x}=\boldsymbol{c_1}}$*;*
2. Do2. *Exchange between intervention and conditioning:*
$$\vdash_g \lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil \lfloor \boldsymbol{c_1}/\boldsymbol{x} \rfloor (\mathit{dsep}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \wedge \textstyle\bigwedge_{\boldsymbol{s} \in S} \mathit{pos}(\boldsymbol{s})) \rightarrow ((\lceil \boldsymbol{c_0}/\boldsymbol{v}, \boldsymbol{c_1}/\boldsymbol{x} \rceil \varphi_0) \leftrightarrow \lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil \varphi_1)$$
*where* $\varphi_1$ *is obtained by replacing every occurrence of* $\boldsymbol{y}|_{\boldsymbol{z}}$ *in* $\varphi_0$ *with* $\boldsymbol{y}|_{\boldsymbol{z}, \boldsymbol{x}=\boldsymbol{c_1}}$*;*
3. Do3 *Introduction/elimination of intervention:*
$$\vdash_g \lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil (\mathit{allnanc}(\boldsymbol{x_1}, \boldsymbol{x}, \boldsymbol{y}) \wedge \lceil \boldsymbol{c_1}/\boldsymbol{x_1} \rceil (\mathit{dsep}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \wedge \mathit{pos}(\boldsymbol{z}))) \rightarrow ((\lceil \boldsymbol{c_0}/\boldsymbol{v} \rceil \varphi) \leftrightarrow \lceil \boldsymbol{c_0}/\boldsymbol{v}, \boldsymbol{c_1}/\boldsymbol{x_1}, \boldsymbol{c_2}/\boldsymbol{x_2} \rceil \varphi)$$
*where* $\mathit{fv}(\varphi) = \{\boldsymbol{y}|_{\boldsymbol{z}}\}$ *and* $\boldsymbol{x} \stackrel{def}{=} \boldsymbol{x_1} :: \boldsymbol{x_2}$*.*

By using the deductive system $\mathbf{AX}^{CP}$, we can derive those rules. Thanks to the modal operators for lazy interventions, our derivation of those rules is partly different from Pearl's [30] in that it does not use diagrams augmented with the intervention arc of the form $F_x \rightarrow x$ (See Appendix D in the full version [25]).

### 7.2 Reasoning About Statistical Adjustment

We present how $\mathbf{AX}^{CP}$ can be used to reason about the correctness of the backdoor adjustment discussed in Section 2 (See Appendix A.6 in the full version [25] for the details of the backdoor adjustment). Fig. 7 shows the derivation of the judgment:

$$\psi_{\mathrm{pre}} \vdash_g (\lceil c/\boldsymbol{x} \rceil c_0 = \boldsymbol{y}) \leftrightarrow (\psi_1 \wedge \psi_2 \wedge \psi_3). \quad (3)$$

This judgment asserts the correctness of the backdoor adjustment in any causal diagram. Recall that $\varphi_{\mathrm{RCT}} \stackrel{\mathrm{def}}{=} (\lceil c/\boldsymbol{x} \rceil c_0 = \boldsymbol{y})$ expresses the RCT and $\varphi_{\mathrm{BDA}} \stackrel{\mathrm{def}}{=} (\psi_1 \wedge \psi_2 \wedge \psi_3)$ expresses the backdoor adjustment. The correctness of the backdoor adjustment ($\varphi_{\mathrm{RCT}} \leftrightarrow \varphi_{\mathrm{BDA}}$) depends on the precondition $\psi_{\mathrm{pre}}$.

By reading the derivation tree in a bottom-up manner, we observe that the proof first converts ($\lceil c/\boldsymbol{x} \rceil c_0 = \boldsymbol{y}$) to a formula to which Eq$_{\mathrm{C}}$ and Eq$_{\mathrm{F}}$ are applicable. Then, the derived axioms Do2 and Do3 in Proposition 2 are used to complete the proof at the leaves of the derivation.

In Section 2, we stated the correctness of the backdoor adjustment in (1) using a simpler requirement $pa(z, x)$ instead of $\psi_{\mathrm{d1}}$ and $\psi_{\mathrm{nanc}}$. We can derive the judgment (1) from (3), thanks to the axioms PaDsep and PaNanc.

The derivation does not mention the data generator $g$ representing the causal diagram $G$. This exhibits that our logic successfully separates the reasoning about the properties of arbitrary causal diagrams from those depending on a specific causal diagram. Once we prove $\psi_{\mathrm{pre}} \vdash_g \varphi_{\mathrm{RCT}} \leftrightarrow \varphi_{\mathrm{BDA}}$ using $\mathbf{AX}^{CP}$, one can claim the correctness of the causal inference ($\varphi_{\mathrm{RCT}} \leftrightarrow \varphi_{\mathrm{BDA}}$) by checking that the re-

**Axioms for $d$-separation**

| | |
|---|---|
| DsepCI | $\vdash_g (\mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \wedge \mathit{pos}(\boldsymbol{z})) \to \boldsymbol{y}\vert_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}} = \boldsymbol{y}\vert_{\boldsymbol{z}}$ |
| DsepSm | $\vdash_g \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \leftrightarrow \mathit{dsep}(\boldsymbol{y},\boldsymbol{x},\boldsymbol{z})$ |
| DsepDc | $\vdash_g \mathit{dsep}(\boldsymbol{x},\boldsymbol{y}\cup\boldsymbol{y}',\boldsymbol{z}) \to (\mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \wedge \mathit{dsep}(\boldsymbol{x},\boldsymbol{y}',\boldsymbol{z}))$ |
| DsepWu | $\vdash_g \mathit{dsep}(\boldsymbol{x},\boldsymbol{y}\cup\boldsymbol{v},\boldsymbol{z}) \to \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}\cup\boldsymbol{v})$ |
| DsepCn | $\vdash_g (\mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \wedge \mathit{dsep}(\boldsymbol{x},\boldsymbol{v},\boldsymbol{z}\cup\boldsymbol{y})) \to \mathit{dsep}(\boldsymbol{x},\boldsymbol{y}\cup\boldsymbol{v},\boldsymbol{z})$ |

**Axioms for $d$-separation with interventions**

| | |
|---|---|
| $\text{Dsep}_{\text{EI}}$ | $\vdash_g (\lceil \boldsymbol{c}/\boldsymbol{z} \rceil \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})) \leftrightarrow \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ |
| $\text{Dsep}_{\text{LI}}$ | $\vdash_g (\lfloor \boldsymbol{c}/\boldsymbol{z} \rfloor \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})) \leftrightarrow \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ |

**Axioms with other causal predicates**

| | |
|---|---|
| Nanc1 | $\vdash_g (\mathit{nanc}(\boldsymbol{x},\boldsymbol{y}) \wedge \mathit{nanc}(\boldsymbol{x},\boldsymbol{z})) \to (f = \boldsymbol{y}\vert_{\boldsymbol{z}} \leftrightarrow \lceil \boldsymbol{c}/\boldsymbol{x} \rceil (f = \boldsymbol{y}\vert_{\boldsymbol{z}}))$ |
| Nanc2 | $\vdash_g \mathit{nanc}(\boldsymbol{x},\boldsymbol{y}) \leftrightarrow \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \mathit{nanc}(\boldsymbol{x},\boldsymbol{y})$ |
| Nanc3 | $\vdash_g \mathit{nanc}(\boldsymbol{x},\boldsymbol{y}) \to \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\emptyset)$ |
| Nanc4 | $\vdash_g (\mathit{nanc}(\boldsymbol{x},\boldsymbol{z}) \wedge \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})) \to \mathit{nanc}(\boldsymbol{x},\boldsymbol{y})$ |
| AllNanc | $\vdash_g \mathit{allnanc}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \to \mathit{nanc}(\boldsymbol{x},\boldsymbol{z})$ |
| PaNanc | $\vdash_g \mathit{pa}(\boldsymbol{x},\boldsymbol{y}) \to \mathit{nanc}(\boldsymbol{y},\boldsymbol{x})$ |
| PaDsep | $\vdash_g \mathit{pa}(\boldsymbol{z},\boldsymbol{x}) \to \lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ |

Fig.6: The additional axioms for $\mathbf{AX}^{CP}$ where $\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}',\boldsymbol{z},\boldsymbol{v} \in \mathsf{CVar}^+$ are disjoint, $\boldsymbol{c} \in \mathsf{dConst}^+$, and $f \in \mathsf{Fsym}$.

$$
\dfrac{
\dfrac{}{\vdash_g \psi_{\text{d1}} \to ((\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_0) \leftrightarrow \psi_1)}\ \text{Do2}
\qquad
\dfrac{\dfrac{}{\vdash_g \psi_{\text{nanc}} \to ((\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_2) \leftrightarrow \psi_2)}\ \text{Nanc3}}{\vdash_g \psi_{\text{d2}} \to ((\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_2) \leftrightarrow \psi_2)}\ \text{Do3}
\qquad
\dfrac{}{\vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_3) \leftrightarrow \psi_3}\ \text{Eq}_{\text{EI}}
}{
\dfrac{\dfrac{\dfrac{\dfrac{\psi_{\text{pre}} \vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_0 \wedge \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_2 \wedge \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \psi_3) \leftrightarrow (\psi_1 \wedge \psi_2 \wedge \psi_3)}{\psi_{\text{pre}} \vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil (\psi_0 \wedge \psi_2 \wedge \psi_3)) \leftrightarrow (\psi_1 \wedge \psi_2 \wedge \psi_3)}\ \text{Distr}_{\text{EI}}^{\wedge}}{\psi_{\text{pre}} \vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil c_0 = (\boldsymbol{y}\vert_{\boldsymbol{z}}(\boldsymbol{z}))\downarrow_{\boldsymbol{y}}) \leftrightarrow (\psi_1 \wedge \psi_2 \wedge \psi_3)}\ \text{Eq}_{\text{C}}, \text{Eq}_{\text{F}}, \text{Eq2}}{\psi_{\text{pre}} \vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil c_0 = (\boldsymbol{y} :: \boldsymbol{z})\downarrow_{\boldsymbol{y}}) \leftrightarrow (\psi_1 \wedge \psi_2 \wedge \psi_3)}\ \text{PD, Eq2}}{\psi_{\text{pre}} \vdash_g (\lceil \boldsymbol{c}/\boldsymbol{x} \rceil c_0 = \boldsymbol{y}) \leftrightarrow (\psi_1 \wedge \psi_2 \wedge \psi_3)}\ \text{MPD, Eq2}
}
$$

Fig.7: Sketch of a derivation tree for the correctness of the backdoor adjustment (Section 2) using $\mathbf{AX}^{CP}$ where $\psi_{\text{pos}} \stackrel{\text{def}}{=} \mathit{pos}(\boldsymbol{z} :: \boldsymbol{x})$, $\psi_{\text{d1}} \stackrel{\text{def}}{=} \lfloor \boldsymbol{c}/\boldsymbol{x} \rfloor \mathit{dsep}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \wedge \psi_{\text{pos}}$, $\psi_{\text{d2}} \stackrel{\text{def}}{=} \lceil \boldsymbol{c}/\boldsymbol{x} \rceil \mathit{dsep}(\boldsymbol{x},\boldsymbol{z},\emptyset) \wedge \psi_{\text{pos}}$, $\psi_{\text{nanc}} \stackrel{\text{def}}{=} \mathit{nanc}(\boldsymbol{x},\boldsymbol{z}) \wedge \psi_{\text{pos}}$, $\psi_{\text{pre}} \stackrel{\text{def}}{=} \psi_{\text{d1}} \wedge \psi_{\text{nanc}}$, $\psi_0 \stackrel{\text{def}}{=} (f = \boldsymbol{y}\vert_{\boldsymbol{z}})$, $\psi_1 \stackrel{\text{def}}{=} (f = \boldsymbol{y}\vert_{\boldsymbol{z},\boldsymbol{x}=\boldsymbol{c}})$, $\psi_2 \stackrel{\text{def}}{=} (c_1 = \boldsymbol{z})$, and $\psi_3 \stackrel{\text{def}}{=} (c_0 = f(c_1)\downarrow_{\boldsymbol{y}})$.

quirement $\psi_{\text{pre}}$ indeed holds for a specific causal diagram $G$.

## 8 Conclusion

We proposed statistical causality language (StaCL) to formally describe and explain the correctness of statistical causal inference. We introduced the notion of causal predicates and Kripke models equipped with data generators. We defined a sound deductive system $\mathbf{AX}^{CP}$ that can deduce all causal effects derived using Pearl's do-calculus. In ongoing and future work, we study the completeness of $\mathbf{AX}$ and $\mathbf{AX}^{CP}$ and develop a decision procedure for $\mathbf{AX}^{CP}$ for automated reasoning.

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